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Analysis of a Symmetrically Loaded Sandwich Cylinder

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Nomenclature

$D_x\,D_y$ = beam flexural stiffnesses per inch of width of orthotropic shell in axial and circumferential directions respectively, in lb
D_{Q_x} = shear stiffness in xz plane per inch of width, lb/in
E_x, E_y = extensional stiffnesses of orthotropic shell in axial and circumferential directions, respectively lb/in
G_c = core shear modulus in xz plane, psi
M_x = moment acting in the x direction, in lb/in
N = tension force acting in the centroidal plane, lb/in
p = pressure acting on cylinder in direction normal to plane of sandwich, psi
u,v,w = displacement in x, y, and z directions, respectively
Q_x = transverse shear force acting in xz plane, lb/in
μ_x,μ_y = Poisson's ratio associated with bending in x and y directions, respectively
μ_x',μ_y' = Poisson's ratios associated with extension in x and y directions, respectively

Introduction

A PROCEDURE for analyzing homogeneous isotropic cylinders loaded symmetrically along the longitudinal axis was presented by Timoshenko [1] In that analysis, it was assumed that shear distortion is negligible, compared with bending distortion However, in the case of a cylinder constructed from a sandwich with a relatively low traverse shear rigidity, the shear distortion may not be negligible Therefore, an analysis is presented in the following paragraphs that includes shear distortion for a symmetrically loaded orthotropic sandwich cylinder The shear deformations are taken into consideration by the same method used by Libove and Batdorf[2] in accounting for shear deformations in a sandwich plate

Derivation of the Differential Equation

As shown in Fig 1, the cylinder is loaded symmetrically with respect to the longitudinal axis Only small deflections

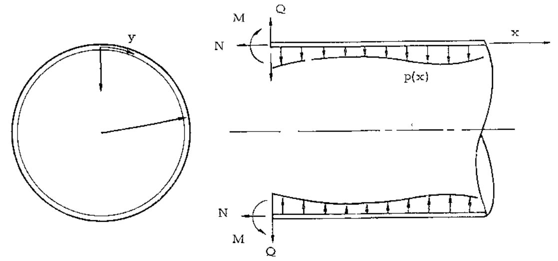

Fig 1 Cylinder subjected to symmetrical external forces along axis

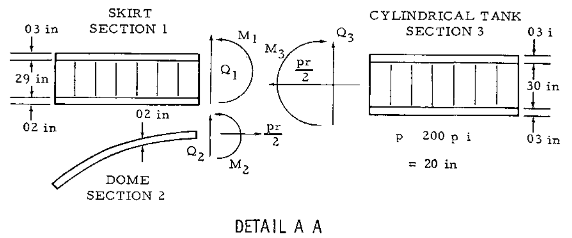

Fig 2 Pressure vessel joint detail

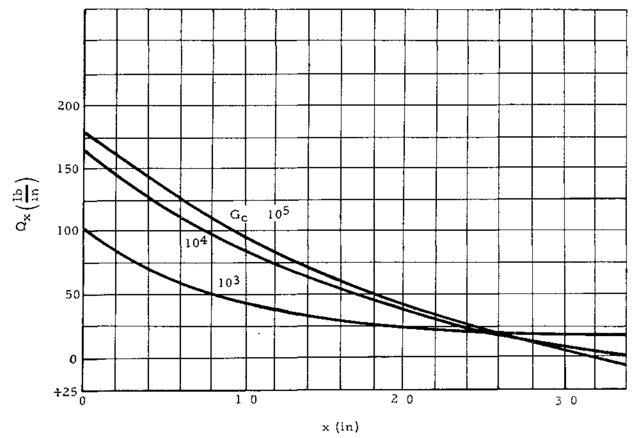

Fig 3 Effects of shear deflection on transverse shear force

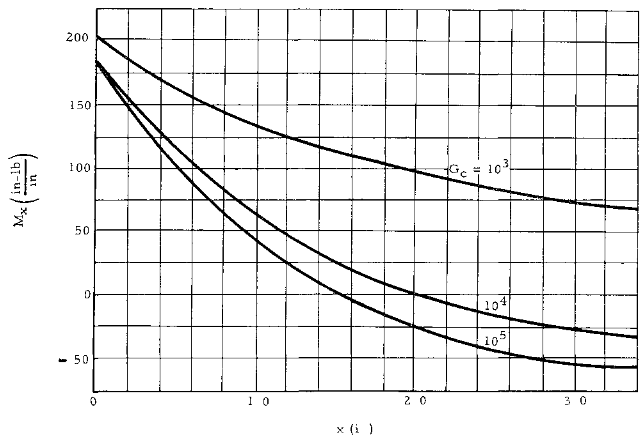

Fig 4 Effects of shear deflection on internal moments

are considered Because of symmetry, the resultant moment in the x direction, M_x, is written as a function of just two dependent variables:

$$M_x = -\frac{D_x}{1-\mu_x\mu_y}\left[\frac{d^2w}{dx^2} - \frac{1}{D_{Q_x}}\frac{dQ_x}{dx}\right] \tag{1}$$

The three equations of equilibrium are

$$(dN_x/dx) = 0 \tag{2}$$

$$N_x\frac{d^2w}{dx^2} + \frac{dQ_x}{dx} + \frac{N_y}{r} + p = 0 \tag{3}$$

$$(dM_x/dx) - Q_x = 0 \tag{4}$$

It can be seen from Eq (2) that the force N_x is constant; therefore $N_x = N_0$

The forces in the middle surface of an orthotropic shell in terms of the deflections are obtained from Ref 3:

$$N_0 = \frac{E_x}{1-\mu_x'\mu_y'}\left[\frac{du}{dx} + \mu_y'\left(-\frac{w}{r}\right)\right] \tag{5}$$

$$N_y = \frac{E_y}{1-\mu_x'\mu_y'}\left[\left(-\frac{w}{r}\right) + \mu_x'\frac{du}{dx}\right] \tag{6}$$

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Table 1 Solution of the differential equation

$\alpha^4 - \beta^4 > 0$	$\alpha^4 = \beta^4$	$\alpha^4 - \beta^4 < 0$
$w = \frac{BM_0 + Q_0}{D(B-A)(4\alpha^2 - A^2)} e^{-Ax} + \frac{AM_0 + Q_0}{D(B-A)(B^2 - 4\alpha^2)} e^{-Bx}$	$w = \left[\frac{Q_0}{Dm\alpha^2} - \frac{M_0}{2D\alpha^2}\right] e^{mx} + \left[\frac{Q_0}{2D\alpha^2} - \frac{M_0}{Dm}\right] xe^{mx}$	$w = \frac{e^{-Rx}}{2D\beta^4}\left[\left(-\beta^2 \cos Px + \frac{R}{P}\beta^2 \sin Px\right)M_0 + \left(\frac{\alpha^2}{P}\sin Px - R\cos Px\right)Q_0\right]$
$M_x = \frac{Q_0[e^{-Ax} - e^{-Bx}]}{B-A} + \frac{M_0[Be^{-Ax} - Ae^{-Bx}]}{B-A}$	$M_x = e^{mx}[M_0 + (Q_0 - mM_0)x]$	$M_x = \frac{e^{-Rx}}{P}[Q_0 \sin Px + M_0(P\cos Px + R\sin Px)]$
$Q_x = \frac{Q_0[-Ae^{-Ax} + Be^{-Bx}]}{B-A} + \frac{M_0[-ABe^{-Ax} + ABe^{-Bx}]}{B-A}$	$Q_x = e^{mx}[Q_0 + (mQ_0 - 2\alpha^2 M_0)x]$	$Q_x = \frac{e^{-Rx}}{P}[Q_0(P\cos Px - R\sin Px) - M_0(R^2 + P^2)\sin Px]$
where	where	where
$A = [2\alpha^2 + 2(\alpha^4 - \beta^4)^{1/2}]^{1/2}$	$m = -(2)^{1/2}\alpha$	$R = (\beta^2 + \alpha^2)^{1/2}$
$B = [2\alpha^2 - 2(\alpha^4 - \beta^4)^{1/2}]^{1/2}$		$P = (\beta^2 - \alpha^2)^{1/2}$

Table 2 Internal forces at joint

G_c, psi	Q_1, lb/in	Q_2, lb/in	Q_3, lb/in	M_1, in lb/in	M_2 in -lb/in	M_3 in -lb/in
10^3	82 6	17 3	−100 0	−132 9	−3 1	204 0
10^4	143 1	20 3	−163 4	−152 2	−4 4	183 4
10^5	158 9	20 8	−180 0	−156 5	−4 6	178 9

By combining Eqs (1–6), the following differential equation results:

$$\left[1 + \frac{N_0}{D_{Q_x}}\right]\frac{d^4w}{dx^4} - \left[\frac{E_y}{D_{Q_x}r^2} + \frac{1 - \mu_x\mu_y}{D_x}N_0\right]\frac{d^2w}{dx^2} + \frac{E_y(1 - \mu_x\mu_y)}{D_x r^2} w = \frac{1 - \mu_x\mu_y}{D_x}\left[\frac{\mu_y}{r}N_0 + p\right] \quad (7)$$

Solution of the Differential Equation

Equation (7) was solved for the case of a long cylinder subjected to a moment M_0 and a transverse shear Q_0, as shown in Fig 1 The normal pressure p and the axial force N_0 were made equal to zero The solution for Eq (7) is shown in Table 1 The constants are defined as follows:

$$\alpha^2 = \frac{E_y}{4D_{Q_x}r^2} \qquad \beta^4 = \frac{E_y(1 - \mu_x\mu_y)}{4D_x r^2} \qquad D = \frac{D_x}{(1 - \mu_x\mu_y)}$$

The influence coefficients for the displacement and rotation of the edge of the shell are

$$[w]_{x=0} = -\left[\frac{(\beta^2 + \alpha^2)^{1/2}}{2\beta^4 D}\right]Q_0 - \left[\frac{1}{2\beta^2 D}\right]M_0 \quad (8)$$

$$\left[\frac{dw}{dx}\right]_{x=0} = \left[\frac{2\alpha^2 + \beta^2}{2D\beta^4}\right]Q_0 + \left[\frac{(\alpha^2 + \beta^2)^{1/2}}{D\beta^2}\right]M_0 \quad (9)$$

Equations (8) and (9) are valid for $\alpha^4 - \beta^4 > 0$, for $\alpha^4 = \beta^4$, and for $\alpha^4 - \beta^4 < 0$

Example

The joint of a sandwich pressure vessel with various shear rigidities is analyzed here to show the effects of shear deflections on the internal forces Figure 2 illustrates the pressure vessel and idealized structural model In analyzing the joint, the following material properties were used: elastic modulus equals 3 5 × 10^6 psi in the x direction and 4 5 × 10^6 psi in the y direction; Poisson's ratio equals 0 15 in both directions The internal pressure is 200 psi The internal forces Q_1, Q_2, Q_3, M_1, M_2, and M_3 were determined for each of three different sandwich core shear moduli G, and the results are shown in Table 2 It can be seen from Table 2 that shear deflections may have an important influence on the internal loads

The variation of the shear force Q_x as a function of x, plotted in Fig 3, is for section 3 of the joint for each value of core shear rigidity The variation of the moment M_x as a function of x is plotted in Fig 4 The shear forces Q_x and moments M_x were obtained from Table 1 It is obvious from Figs 3 and 4 that the shear and moment at the edge of a cylinder die out faster if the shear rigidity is large

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Deformations and Stresses in an Axially Restrained Beam

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Nomenclature

a_0 = amplitude of lateral deflection of unrestrained beam due to lateral load and thermal gradients

a_1 = amplitude of lateral deflection of restrained beam

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